

# Flavor and CP Violation Induced by Atmospheric Neutrino Mixing

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Neutrino oscillation experiments suggest existence of new flavor-violating interactions in high energy scale. It may be possible to probe them by the flavor- and CP-violating processes in leptons and hadrons in the supersymmetric (SUSY) models. In this article the third-generation flavor violation is reviewed in the SUSY seesaw model and SUSY SU(5) GUT with the right-handed neutrinos.

## 1. Introduction

After discovery of the atmospheric neutrino oscillation by the superKamiokande experiment in 98' [1], it is found that the lepton sector has much different flavor structure from the quark sector [2][3]. The seesaw mechanism is the most promising model to explain the tiny neutrino masses and the neutrino oscillation [4]. However, the observed mixing angles between the first and second and between second and third generations are almost maximal, and those are different from the naive expectation in the extension of the seesaw mechanism to the grand unified theories (GUTs). Various attempts to understand those mismatches between quark and lepton mixing angles have been made so far.

We may find new clues to understand the origin of the neutrino masses in the future precision experiments of hadrons and leptons. In the supersymmetric (SUSY) models, the flavor- and CP-violating phenomena in hadron and lepton physics are windows to probe flavor structure in the high energy physics, such as the seesaw mechanism and the GUTs. The imprints may be generated in the SUSY-breaking slepton or squark mass terms if the SUSY-breaking terms are generated at the higher than the energy scale. In this case the SUSY-breaking terms induce the sizable effect to the flavor- and CP-violating processes in hadrons and leptons, since they are suppressed by only the SUSY-breaking scale, not the energy scale responsible to the flavor violation such as the right-handed neutrino mass or GUT scale [5].

In the SUSY seesaw mechanism the radiative correction by the neutrino Yukawa coupling induces the left-handed slepton mixing even if the soft SUSY-breaking terms are universal at the GUT or Planck scale. This lepton-flavor violation (LFV) leads to the radiative LFV decay of tau and muon [6]. If the SUSY seesaw model is extended to the SUSY GUTs, the right-handed down-type squarks also have the flavor-violating SUSY-breaking mass terms since they are embedded in common SU(5) multiplets with the left-handed leptons [7][8]. This may lead to new flavor- and CP-violating source.

The Belle and BaBar experiments in the KEK and SLAC  $B$  factories, give new information about the flavor violation of the third generations. The CKM dominance in the flavor- and CP-violating hadron phenomena is almost confirmed by the experiments. However, the Belle experiment reports that the CP asymmetry in  $B_d \rightarrow \phi K_s$  is  $3.5\sigma$  deviated from the Standard-Model (SM) prediction [9]. At present the BaBar experiment does not observe such a large deviation, and the combined result is not yet significant [10]. However, the Belle's result might be a signature of the new physics. In addition to them, the experiments are improving bounds on the LFV tau decay modes by about one order of magnitude [11]. Now the physics in high luminosity  $B$  factories, whose integrate luminosities may reach to the order of  $ab^{-1}$ , is discussed. If these super  $B$  factories are constructed, we may get new information about the origin of the neutrino masses.

In this article, we review the third generation

flavor violation in leptons and hadrons by the neutrino Yukawa interaction in the SUSY seesaw mechanism and the extension to SUSY GUTs, which will be probed in the super  $B$  factories. The atmospheric neutrino experiments suggest the existence of large flavor violation between the second and the third generations. The LFV tau decay is a good probe to the neutrino Yukawa coupling responsible to the atmospheric neutrino oscillation.

The SUSY GUTs may have rich flavor physics in the hadron and lepton sectors as mentioned above. However, since the flavor- and CP-violating phenomena are indirect probes to the new physics, it is important to take correlation and consistency among various processes. It is argued that the right-handed sbottom and strange mixing may induce the sizable deviation for  $B_d \rightarrow \phi K_s$  from the SM prediction in this model [8]. On the other hand, it is pointed out that the deviation is strongly correlated with the chromoelectric dipole moment (CEDM) of the strange quark, and the size of the deviation is limited by the experimental bound on the Mercury EDM [12].

In next section we review the flavor structure in the minimal SUSY seesaw model, and the prediction for the LFV tau decay modes and the sensitivities in the future experiments are discussed in Section 3. We review the flavor structure in the SUSY SU(5) GUT with the right-handed neutrinos in Section 4, and the EDMs in the SUSY GUTs is discussed in Section 5. We show the correlation between the EDM of Mercury and the CP asymmetry in  $B_d \rightarrow \phi K_s$  there. Section 6 is devoted to summary.

## 2. Minimal SUSY Seesaw Model

The seesaw mechanism is the most fascinating model to explain the small neutrino masses in a natural and economical way. In this mechanism the superheavy right-handed neutrinos  $\bar{N}$  are introduced, and thus, the supersymmetry is required to stabilize the hierarchical structure. In this section we review the flavor structure in the SUSY-breaking terms in the minimal SUSY seesaw model.

In the minimal SUSY seesaw model only three

additional heavy singlet neutrino superfields are introduced. The relevant leptonic part of its superpotential is

$$W_{\text{seesaw}} = f_{ij}^\nu L_i \bar{N}_j \bar{H}_f + f_{ij}^l \bar{E}_i L_j H_f + \frac{1}{2} M_{ij} \bar{N}_i \bar{N}_j \quad (1)$$

where the indexes  $i, j$  run over three generations and  $M_{ij}$  is the heavy singlet neutrino mass matrix. In addition to the three charged lepton masses, this superpotential has eighteen physical parameters, including six real mixing angles and six CP-violating phases, because the Yukawa coupling and the Majorana mass matrices are given after removing unphysical phases as

$$f_{ij}^l = f_{li} \delta_{ij}, \quad (2)$$

$$f_{ij}^\nu = X_{ik}^* f_{\nu k} e^{-i\varphi_{\nu k}} W_{kj}^* e^{-i\bar{\varphi}_{\nu k}}, \quad (3)$$

$$M_{ij} = \delta_{ij} M_{N_k}, \quad (4)$$

Here,  $\sum_i \varphi_{\nu i} = 0$  and  $\sum_i \bar{\varphi}_{\nu i} = 0$ , and  $W$  and  $X$  are unitary matrices with one phase. Nine parameters associated with the heavy-neutrino sector cannot be measured in a direct way. The exception is the baryon number in the universe if leptogenesis is right [13].

At low energies the effective theory after integrating out the right-handed neutrinos is given by the effective superpotential

$$W_{\text{eff}} = f_{li} \bar{E}_i L_i H_f + \frac{1}{2v^2 \sin^2 \beta} (m_\nu)_{ij} (L_i \bar{H}_f) (L_j \bar{H}_f), \quad (5)$$

where we work in a basis in which the charged lepton Yukawa couplings are diagonal. The second term in (5) leads to the light neutrino masses and mixings. The explicit form of the small neutrino mass matrix  $(m_\nu)$  is given by

$$(m_\nu)_{ij} = \sum_k \frac{f_{ik}^\nu f_{jk}^\nu}{M_{N_k}} v^2 \sin^2 \beta. \quad (6)$$

The light neutrino mass matrix  $(m_\nu)$  is symmetric, with nine parameters, including three real mixing angles and three CP-violating phases. It can be diagonalized by a unitary matrix  $Z$  as

$$Z^T m_\nu Z = m_\nu^D. \quad (7)$$

By redefinition of fields one can rewrite  $Z \equiv UP$ , where  $P \equiv \text{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$  and  $U$  is the MNS matrix, with the three real mixing angles and the remaining CP-violating phase.

If the SUSY-breaking terms are generated above the right-handed neutrino mass scale, the renormalization effects may induce sizable LFV slepton mass terms, which leads to the LFV charged lepton decay. If the SUSY-breaking parameters at the GUT scale or the Planck scale are universal, off-diagonal components in the left-handed slepton mass matrix ( $m_L^2$ ) and the trilinear slepton coupling ( $A_e$ ) take the approximate forms

$$\begin{aligned} (\delta m_L^2)_{ij} &\simeq -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)H_{ij}, \\ (\delta A_e)_{ij} &\simeq -\frac{1}{8\pi^2}A_0 f_{ei} H_{ij}, \end{aligned} \quad (8)$$

where  $i \neq j$ , and the off-diagonal components of the right-handed slepton mass matrix are suppressed. Here, the Hermitian matrix  $H$ , whose diagonal terms are real and positive, is defined in terms of  $f_\nu$  and the heavy neutrino masses  $M_{N_k}$  by

$$H_{ij} = \sum_k f_{ik}^{\nu*} f_{jk}^\nu \log \frac{M_{GUT}}{M_{N_k}}. \quad (9)$$

Here we take  $M_{GUT}$  the GUT scale for simplicity. In Eq. (8) the parameters  $m_0$  and  $A_0$  are the universal scalar mass and trilinear coupling at the GUT scale. We ignore terms of higher order in  $f_l$ , assuming that  $\tan\beta$  is not extremely large.

The Hermitian matrix  $H$  has nine parameters including three phases, which are clearly independent of the parameters in  $(m_\nu)$ . Thus two matrices  $(m_\nu)$  and  $H$  together provide the required eighteen parameters, including six CP-violating phases, by which we can parameterize the minimal SUSY seesaw model [14].

The off-diagonal terms,  $H_{ij} (i \neq j)$ , are related to the LFV  $l_i - l_j$  transition, and they are related to the LFV charged lepton decay. On the other hand, our abilities to measure three phases in the Hermitian matrix  $H$  are limited, in addition to the Majorana phases  $e^{i\phi_1}$  and  $e^{i\phi_2}$ . Only a phase in  $H$  might be determined by T-odd asymmetries in  $\tau \rightarrow 3l$  or  $\mu \rightarrow 3e$  [15], since they are pro-

portional to a Jaroskog invariant obtainable from  $H$ ,

$$J = \text{Im}[H_{12}H_{23}H_{31}]. \quad (10)$$

In order to determine other two phases in  $H$ , the asymmetries, which come from interference between phases in  $H$  and  $(m_\nu)$ , have to be measured. A possibility to determine them might be the EDMs of the charged leptons [16]. The threshold correction due to non-degeneracy of the right-handed neutrino masses might enhance the EDMs of the charged leptons. They depend on all of the phases in  $(m_\nu)$  and  $H$  in non-trivial ways.

### 3. LFV Tau Decay in Minimal SUSY Seesaw Model

As explained in the previous section we show that the LFV charged lepton decay gives information about the minimal SUSY seesaw model, which is independent of the neutrino oscillation experiments. In this section we demonstrate it by considering the LFV tau decay.

First, we review the LFV charged lepton decay in the SUSY SM. In the SUSY models, the LFV processes of the charged leptons are radiative one due to the  $R$  parity. Thus, the largest LFV tau decay processes are  $\tau \rightarrow \mu\gamma$  or  $\tau \rightarrow e\gamma$ . Other processes are suppressed by order of  $\alpha$ . They are discussed later. The effective operators relevant to  $l \rightarrow l'\gamma$  are flavor-violating dipole moment operators,

$$\begin{aligned} H_{\text{eff}} &= \sum_{l>l'} \frac{4G_F}{\sqrt{2}} m_l \left[ A_R^{ll'} \bar{l} \sigma^{\mu\nu} P_R l' + A_L^{ll'} \bar{l} \sigma^{\mu\nu} P_L l' \right] \\ &+ h.c., \end{aligned} \quad (11)$$

where  $P_{L/R} = (1 \mp \gamma_5)/2$ , and the branching ratios are given as

$$\begin{aligned} Br(l \rightarrow l'\gamma) &= 384\pi^2 (|A_R^{ll'}|^2 + |A_L^{ll'}|^2) \\ &\times Br(l \rightarrow l' \nu_l \bar{\nu}_{l'}). \end{aligned} \quad (12)$$

Here,  $Br(\tau \rightarrow \mu(e)\nu_\tau \bar{\nu}_{\mu(e)}) \simeq 0.17$  and  $Br(\mu \rightarrow e\nu_\mu \bar{\nu}_e) = 1$ . The coefficients in Eq. (11) are approximately given as

$$A_R^{\tau l'} = \frac{\sqrt{2}e}{4G_F} \frac{\alpha_Y}{4\pi} \frac{\tan\beta}{m_{SUSY}^2} \left[ -\frac{1}{120} \delta_{\tau l'}^R \right], \quad (13)$$

$$A_L^{\tau l'} = \frac{\sqrt{2}e}{4G_F} \frac{\alpha_2}{4\pi} \frac{\tan\beta}{m_{SUSY}^2} \left[ \left( \frac{1}{30} + \frac{t_W^2}{24} \right) \delta_{\tau l'}^L \right], \quad (14)$$

$$A_R^{\mu e} = \frac{\sqrt{2}e}{4G_F} \frac{\alpha_Y}{4\pi} \frac{\tan\beta}{m_{SUSY}^2} \left[ -\frac{1}{120} \delta_{\mu e}^R + \frac{1}{120} \delta_{\mu\tau}^R \delta_{\tau e}^R - \frac{1}{60} \frac{m_\tau}{m_\mu} \delta_{\mu\tau}^L \delta_{\tau e}^R \right], \quad (15)$$

$$A_L^{\mu e} = \frac{\sqrt{2}e}{4G_F} \frac{\alpha_2}{4\pi} \frac{\tan\beta}{m_{SUSY}^2} \left[ \left( \frac{1}{30} + \frac{t_W^2}{24} \right) \delta_{\mu e}^L - \left( \frac{1}{80} + \frac{7t_W^2}{240} \right) \delta_{\mu\tau}^L \delta_{\tau e}^L - \frac{t_W^2}{60} \frac{m_\tau}{m_\mu} \delta_{\mu\tau}^R \delta_{\tau e}^L \right], \quad (16)$$

assuming for simplicity that all SUSY particle masses are the same as  $m_{SUSY}$  and  $\tan\beta \gg 1$ . Here,  $t_W \equiv \tan\theta_W$ , where  $\theta_W$  is the Weinberg angle, and the mass insertion parameters are given as

$$\delta_{ij}^R = \left( \frac{(m_E^2)_{ij}}{m_{SUSY}^2} \right), \quad \delta_{ij}^L = \left( \frac{(m_L^2)_{ij}}{m_{SUSY}^2} \right), \quad (17)$$

where  $(m_E^2)$  is the right-handed slepton mass matrix. When the slepton mass matrices are non-vanishing for both  $(1, 3)$  and  $(2, 3)$  components,  $\mu \rightarrow e\gamma$  is generated via stau exchange. Especially, if both the left-handed and right-handed mixing are sizable, the branching ratio is enhanced by  $(m_\tau/m_\mu)^2$  compared with a case that only left-handed or right-handed mixing angles are non-vanishing. The off-diagonal components in  $(A_e)_{ij}$  are sub-dominant in these processes since the contribution is not proportional to  $\tan\beta$ .

We list constraints on  $\delta_{ij}^R$  and  $\delta_{ij}^L$  from current experimental bounds on  $Br(\tau \rightarrow \mu(e)\gamma)$ , which are derived by the Belle experiment [11], and  $Br(\mu \rightarrow e\gamma)$  in Table 1. In this table, we take  $\tan\beta = 10$  and  $m_{SUSY} = 100\text{GeV}$  and  $300\text{GeV}$ . The constraints from  $\mu \rightarrow e\gamma$  on the slepton mixings are quite stringent. On the other hand, the current bounds on the LFV tau decay modes give sizable constraints on  $|\delta_{\tau\mu}^L|$  and  $|\delta_{\tau e}^L|$ , independently. Furthermore, while the current constraint on  $|\delta_{\mu\tau}^L \delta_{\tau e}^L|$  from the LFV tau decay is weaker than that from the LFV muon decay, the improvement of the LFV tau decay modes by about an

order of magnitude will give a competitive bound  $|\delta_{\mu\tau}^L \delta_{\tau e}^L|$  to that from the LFV muon decay.

Now we present sensitivity of the future experiments for the LFV tau decay in the minimal SUSY seesaw model. We take two different limits of the parameter matrix  $H$  given in Eq. (9), of the form [14]

$$H_1 = \begin{pmatrix} a & 0 & 0 \\ 0 & b & d \\ 0 & d^\dagger & c \end{pmatrix}, \quad (18)$$

and

$$H_2 = \begin{pmatrix} a & 0 & d \\ 0 & b & 0 \\ d^\dagger & 0 & c \end{pmatrix}, \quad (19)$$

where  $a, b, c$  are real and positive, and  $d$  is a complex number. The non-vanishing  $(2, 3)$  component in  $H_1$  leads to  $\tau \rightarrow \mu\gamma$  while the  $(1, 3)$  component in  $H_2$  does to  $\tau \rightarrow e\gamma$ .

In the above Ansatz, we take  $H_{12} = 0$  and  $H_{13}H_{32} = 0$  because these conditions suppress  $Br(\mu \rightarrow e\gamma)$ . It is found from the numerical calculation that  $Br(\mu \rightarrow e\gamma)$  is suppressed in a broad range of parameters with the chosen forms  $H_1$  and  $H_2$ . From a viewpoint of the model-building, the matrix  $H_1$  is favored since it is easier to explain the large mixing angles observed in the atmospheric and solar neutrino oscillation experiments by structure of the Yukawa coupling  $f_\nu$ . If we adopt  $H_2$ , we might have to require some conspiracy between  $f_\nu$  and  $M_N$ . However, from a viewpoint of the bottom-up approach, we can always find parameters consistent with the observed neutrino mixing angles for both  $H_1$  and  $H_2$ , as explained in the previous section.

In Fig. 1 we show  $Br(\tau \rightarrow \mu\gamma)$  for the ansatz  $H_1$  and  $Br(\tau \rightarrow e\gamma)$  for  $H_2$  as functions of the lightest stau mass. We take the SU(2) gaugino mass to be  $200\text{ GeV}$ ,  $A_0 = 0$ ,  $\mu > 0$ , and  $\tan\beta = 30$  for the SUSY-breaking parameters in the SUSY SM. We sample the parameters in  $H_1$  or  $H_2$  randomly in the range  $10^{-2} < a, b, c, |d| < 10$ , with distributions that are flat on a logarithmic scale. Also, we require the Yukawa coupling-squared to be smaller than  $4\pi$ , so that  $f_\nu$  remains perturbative up to  $M_G$ .

In order to fix the SUSY seesaw model, we take the light neutrino parameters as  $\Delta m_{32}^2 = 3 \times 10^{-3}$

$m_{SUSY}$	$ \delta_{\tau\mu}^L $	$ \delta_{\tau e}^L $	$ \delta_{\mu e}^L $	$ \delta_{\mu\tau}^L \delta_{\tau e}^L $	$ \delta_{\mu\tau}^R \delta_{\tau e}^L $
100GeV	$2 \times 10^{-2}$	$2 \times 10^{-2}$	$4 \times 10^{-5}$	$1 \times 10^{-4}$ ( $3 \times 10^{-4}$ )	$2 \times 10^{-5}$ ( $7 \times 10^{-3}$ )
300GeV	$2 \times 10^{-1}$	$2 \times 10^{-1}$	$4 \times 10^{-4}$	$9 \times 10^{-4}$ ( $3 \times 10^{-2}$ )	$2 \times 10^{-4}$ ( $6 \times 10^{-1}$ )
$m_{SUSY}$	$ \delta_{\tau\mu}^R $	$ \delta_{\tau e}^R $	$ \delta_{\mu e}^R $	$ \delta_{\mu\tau}^R \delta_{\tau e}^R $	$ \delta_{\mu\tau}^L \delta_{\tau e}^R $
100GeV	$3 \times 10^{-1}$	$3 \times 10^{-1}$	$9 \times 10^{-4}$	$9 \times 10^{-4}$ ( $1 \times 10^{-1}$ )	$2 \times 10^{-5}$ ( $7 \times 10^{-3}$ )
300GeV	3	3	$8 \times 10^{-3}$	$8 \times 10^{-3}$ (9)	$2 \times 10^{-4}$ ( $6 \times 10^{-1}$ )

Table 1

Constraints on  $\delta_{ll'}^R$  and  $\delta_{ll'}^L$  from current experimental bounds on  $Br(l \rightarrow l'\gamma)$ . Here, we use the result for the LFV tau decay search by the Belle experiment [11]. We take  $\tan\beta = 10$  and  $m_{SUSY} = 100\text{GeV}$  and  $300\text{GeV}$ . The numbers in parentheses are derived from constraints on  $|\delta_{\tau\mu}^{(L/R)}|$  and  $|\delta_{\tau e}^{(L/R)}|$ .

$\text{eV}^2$ ,  $\Delta m_{21}^2 = 4.5 \times 10^{-5} \text{ eV}^2$ ,  $\tan^2 \theta_{23} = 1$ ,  $\tan^2 \theta_{12} = 0.4$ ,  $\sin \theta_{13} = 0.1$  and  $\delta = \pi/2$ . The Majorana phases  $e^{i\phi_1}$  and  $e^{i\phi_2}$  are taken randomly. In Fig. 1 we assume the normal hierarchy for the light neutrino mass spectrum, however, it is found as expected that the branching ratios are insensitive to the structure of the light neutrino mass matrix.

Current experimental bounds for branching ratios of the LFV tau decay modes are derived in the Belle experiment, and  $Br(\tau \rightarrow \mu(e)\gamma) < 3.2(3.6) \times 10^{-7}$  [11]. These results already exclude a fraction of the parameter space of the minimal SUSY seesaw model. These bounds may be improved to about  $10^{-8}$  in the super  $B$  factories if the signals are not observed [11].<sup>1</sup>

Finally, we discuss other tau LFV processes in the minimal SUSY seesaw models. In a broader parameter space,  $\tau \rightarrow \mu(e)\gamma$  is the largest tau LFV processes, unless it is suppressed by some accidental cancellation or much heavier SUSY particle masses. The LFV tau decay modes to three leptons are dominantly induced by the photon-penguin contributions, and they are correlated

with  $\tau \rightarrow \mu(e)\gamma$  as

$$Br(\tau \rightarrow \mu 2e)/Br(\tau \rightarrow \mu\gamma) \simeq 1/94, \quad (20)$$

$$Br(\tau \rightarrow 3\mu)/Br(\tau \rightarrow \mu\gamma) \simeq 1/440, \quad (21)$$

$$Br(\tau \rightarrow 3e)/Br(\tau \rightarrow e\gamma) \simeq 1/94, \quad (22)$$

$$Br(\tau \rightarrow e 2\mu)/Br(\tau \rightarrow e\gamma) \simeq 1/440. \quad (23)$$

The LFV tau decay modes into pseudo-scalar mesons tend to be smaller than those to three leptons since the branching ratios are not proportional to  $\tan^2 \beta$ .

When sleptons are much heavier than the weak scale,  $Br(\tau \rightarrow \mu(e)\gamma)$  is suppressed. In this case,  $\tau \rightarrow \mu(e)2\mu$  and  $\tau \rightarrow \mu(e)\eta$  induced by Higgs boson exchange become relatively important [19]. The LFV anomalous Yukawa coupling for the Higgs bosons is generated by the radiative correction, and it is not suppressed by powers of the slepton masses. While these processes are suppressed by a small Yukawa coupling constant for muon or strange quark, they may have sizable branching ratios when  $\tan\beta$  is large since the branching ratios are proportional to  $\tan^6 \beta$ . When  $\delta_{\tau\mu}^L$  is non-vanishing, the approximate formula for  $Br(\tau \rightarrow 3\mu)$  is given as

$$Br(\tau \rightarrow 3\mu) = \frac{m_\mu^2 m_\tau^2 \epsilon_2^2 |\delta_{\tau\mu}^L|^2}{8 \cos^6 \beta} Br(\tau \rightarrow \mu\nu_\tau \bar{\nu}_\mu) \times \left[ \left( \frac{\sin(\alpha - \beta) \cos \alpha}{M_{H^0}^2} - \frac{\cos(\alpha - \beta) \sin \alpha}{M_{h^0}^2} \right)^2 \right]$$

<sup>1</sup>If sleptons are found in the future collider experiments, the tau flavor violation might be found in the signal. The cross sections for  $e^+e^- (\mu^+\mu^-) \rightarrow \tilde{l}^+\tilde{l}^- \rightarrow \tau^\pm\mu^\mp (\tau^\pm e^\mp) + X$  are suppressed by at most the mass difference over the widths for the sleptons [17]. The search for these processes in the collider experiments has more sensitivity for a small  $\tan\beta$  region compared with the search for the LFV tau decay [18].

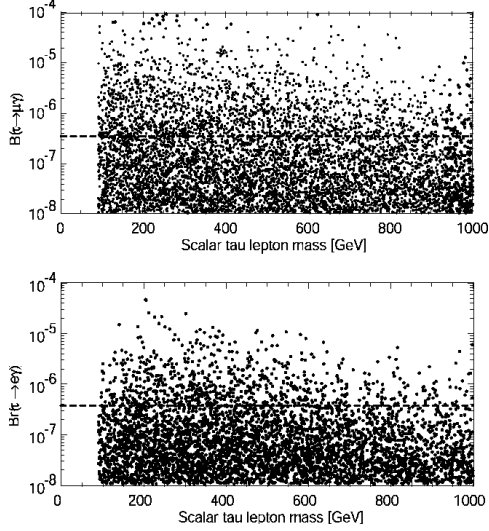


Figure 1.  $Br(\tau \rightarrow \mu\gamma)$  for  $H_1$  and  $Br(\tau \rightarrow e\gamma)$  for  $H_2$ . The input parameters are given in text.

$$\begin{aligned}
 & + \frac{\sin^2 \beta}{M_{A^0}^4} \Big] \\
 & \simeq 3.8 \times 10^{-7} \times |\delta_{\tau\mu}^L|^2 \\
 & \times \left( \frac{\tan \beta}{60} \right)^6 \left( \frac{M_{A^0}}{100 \text{ GeV}} \right)^{-4}, \quad (24)
 \end{aligned}$$

and  $Br(\tau \rightarrow \mu\eta)$  is roughly five times larger than  $Br(\tau \rightarrow 3\mu)$  though it depends on the anomalous coupling of the Higgs bosons to the bottom and strange quark [20]. Here,  $\epsilon_2$  is a function of the SUSY particle masses. We take a limit of large  $\tan \beta$  and equal SUSY-breaking mass parameters in the last step in Eq. (24). Notice that  $\tau \rightarrow \mu\gamma$  also has a comparable branching ratio to them even if the Higgs mediation is dominated, since the Higgs loop diagram is enhanced by the tau Yukawa coupling constant.

#### 4. SUSY SU(5) GUT with Right-Handed Neutrinos

Let us extend the minimal SUSY seesaw model to the SUSY SU(5) GUT. In this model, doublet leptons and right-handed down-type quarks

are embedded in common **5**-dimensional multiplets, while doublet quarks, right-handed up-type quarks, and right-handed charged leptons are in the **10**-dimensional ones. Thus, the neutrino Yukawa coupling induces the right-handed down-type squark mixing, and this leads to rich flavor and CP violating phenomena in hadrons.

In this paper the minimal structure in the Yukawa coupling is assumed for simplicity. The Yukawa coupling for quarks and leptons and the Majorana mass term for the right-handed neutrinos in this model are given as

$$\begin{aligned}
 W_{\text{SU}(5)} = & \frac{1}{4} f_{ij}^u \Psi_i \Psi_j H + \sqrt{2} f_{ij}^d \Psi_i \Phi_j \bar{H} \\
 & + f_{ij}^\nu \Phi_i \bar{N}_j H + \frac{1}{2} M_{ij} \bar{N}_i \bar{N}_j, \quad (25)
 \end{aligned}$$

where  $\Psi$  and  $\Phi$  are **10**- and  $\bar{\mathbf{5}}$ -dimensional multiplets, respectively, and  $H$  ( $\bar{H}$ ) is a **5**- ( $\bar{\mathbf{5}}$ -) dimensional Higgs multiplet.

After removing the unphysical phases, the Yukawa coupling constants and Majorana masses in Eq. (25) are given as follows,

$$f_{ij}^u = V_{ki} f_{u_k} e^{i\varphi_{u_k}} V_{kj}, \quad (26)$$

$$f_{ij}^d = f_{di} \delta_{ij}, \quad (27)$$

$$f_{ij}^\nu = e^{i\varphi_{di}} X_{ik}^* f_{\nu_k} e^{-i\varphi_{\nu_k}} W_{kj}^* e^{-i\bar{\varphi}_{\nu_k}}, \quad (28)$$

$$M_{ij} = M_{Ni} \delta_{ij}, \quad (29)$$

where  $\sum_i \varphi_{fi} = 0$  ( $f = u, d, \nu$ ) and  $\sum_i \bar{\varphi}_{fi} = 0$ . Each unitary matrices  $X$ ,  $V$ , and  $W$  have only a phase, again. Here,  $\varphi_u$  and  $\varphi_d$  are CP-violating phases inherent in the SUSY SU(5) GUT. The unitary matrix  $V$  is the CKM matrix in the extension of the SM to the SUSY SU(5) GUT.

The colored-Higgs multiplets  $H_c$  and  $\bar{H}_c$  are introduced in  $H$  and  $\bar{H}$  as SU(5) partners of the Higgs doublets in the SUSY SM, respectively<sup>2</sup>, and they have new flavor-violating interactions. Eq. (25) is represent by the fields in the SUSY SM as follows,

$$W_{\text{SU}(5)} = W_{\text{SUSYSM}+\bar{N}} + \frac{1}{2} V_{ki} f_{u_k} e^{i\varphi_{u_k}} V_{kj} Q_i Q_j H_c$$

<sup>2</sup>While the proton decay by the dimension-five operator, which is induced by the colored-Higgs exchange, is a serious problem in the minimal SUSY SU(5) GUT [22], it depends on the structure of the Higgs sector [23]. In this article, we ignore the proton decay while we adopt the minimal structure of the Higgs sector.

$$\begin{aligned}
& + f_{u_i} V_{ij} e^{i\varphi_{d_j}} \bar{U}_i \bar{E}_j H_c + f_{d_i} e^{-i\varphi_{d_i}} Q_i L_i \bar{H}_c \\
& + e^{-i\varphi_{u_i}} V_{ij}^* f_{d_j} \bar{U}_i \bar{D}_j \bar{H}_c + e^{i\varphi_{d_i}} X_{ij}^* f_{\nu_j} \bar{D}_i \bar{N}_j H_c.
\end{aligned} \tag{30}$$

Here, the superpotential in the SUSY SM with the right-handed neutrinos is

$$\begin{aligned}
W_{\text{SUSY SM}+\bar{N}} &= V_{ji} f_{u_j} Q_i \bar{U}_j H_f + f_{d_i} Q_i \bar{D}_i \bar{H}_f \\
&+ W_{\text{seesaw}}
\end{aligned} \tag{31}$$

The flavor-violating interactions absent in the SUSY SM emerge in the SUSY SU(5) GUT due to existence of the colored-Higgs multiplets.

If the SUSY-breaking terms in the SUSY SM are generated by interactions above the colored-Higgs mass, such as in the supergravity, the sfermion mass terms may get sizable corrections by the colored-Higgs interactions. Here we assume the minimal supergravity scenario again. In this case, the flavor-violating SUSY-breaking mass terms at low energy are induced by the radiative correction, and they are approximately given as

$$\begin{aligned}
(\delta m_Q^2)_{ij} &\simeq -\frac{2}{(4\pi)^2} (3m_0^2 + A_0^2) \\
&\quad V_{ki}^* f_{u_k}^2 V_{kj} \\
&\quad (3 \log \frac{M_G}{M_{GUT}} + \log \frac{M_{GUT}}{M_{SUSY}}), \\
(\delta m_U^2)_{ij} &\simeq -\frac{4}{(4\pi)^2} (3m_0^2 + A_0^2) \\
&\quad e^{i\varphi_{u_i}} V_{ik} f_{d_k}^2 V_{jk}^* e^{-i\varphi_{u_j}} \\
&\quad \log \frac{M_G}{M_{GUT}}, \\
(\delta m_D^2)_{ij} &\simeq -\frac{2}{(4\pi)^2} (3m_0^2 + A_0^2) \\
&\quad e^{-i\varphi_{d_i}} X_{ik} f_{\nu_k}^2 X_{jk}^* e^{i\varphi_{d_j}} \\
&\quad \log \frac{M_G}{M_{GUT}}, \\
(\delta m_L^2)_{ij} &\simeq -\frac{2}{(4\pi)^2} (3m_0^2 + A_0^2) \\
&\quad X_{ik} f_{\nu_k} e^{i\varphi_{\nu_k}} W_{kl} \\
&\quad W_{ml}^* e^{-i\varphi_{\nu_m}} f_{\nu_m} X_{jm}^* \\
&\quad \log \frac{M_G}{M_{N_i}},
\end{aligned}$$

$$\begin{aligned}
(\delta m_E^2)_{ij} &\simeq -\frac{6}{(4\pi)^2} (3m_0^2 + A_0^2) \\
&\quad e^{-i\varphi_{d_i}} V_{ki}^* f_{u_k}^2 V_{kj} e^{i\varphi_{d_j}} \\
&\quad \log \frac{M_G}{M_{GUT}},
\end{aligned} \tag{32}$$

where  $i \neq j$ . Here,  $M_{GUT}$  and  $M_G$  are the GUT scale and the reduced Planck scale, respectively. In the SUSY SM with the right-handed neutrinos, the flavor-violating structures appear only in the left-handed squark and left-handed slepton mass matrices. On the other hand, in the SUSY SU(5) GUT, other sfermions may also have sizable flavor violation. In particular, the CP-violating phases inherent in the SUSY SU(5) GUT appear in  $(m_U^2)$ ,  $(m_D^2)$ , and  $(m_E^2)$  [21].

In this section we assume the minimal structure for the Yukawa coupling constants given in Eq. (25). The  $b/\tau$  mass ratio may be explained by it while the down-type quark and charged lepton masses in the first and second generations are not compatible. The modification of the Yukawa sector, such as introduction of the higher-dimensional operators, the vector-like matters, or the complicate Higgs structure, may change the low-energy prediction for the flavor violation, especially, between the first and second generations. Thus, we will concentrate on the transition between the second and third generations in the following. We assume that the Yukawa coupling constants, including the neutrino one, are hierarchical, and that the extra interaction gives negligible contribution to the transition between the second and third generations.

The neutrino-induced off-diagonal terms for the sfermion masses are  $(m_D^2)_{ij}$  and  $(m_L^2)_{ij}$  ( $i \neq j$ ). Let us demonstrate the good correlation between  $(m_D^2)_{23}$  and  $(m_L^2)_{23}$ . The non-trivial structure in the right-handed neutrino mass matrix may dilute the correlation as in Eq. (32). However, if the right-handed neutrino masses are hierarchical, the correlation is expected to be good, as will be shown.

We use two-generation model for simplicity [15], ignoring the first generation. Here, we adopt the parameterization for the neutrino sector by Casas and Ibarra [24]. In this parametrization the neutrino sector can be parametrized by

the left-handed ( $m_{\nu_i}$ ) and right-handed neutrino masses ( $M_{N_i}$ ), the MNS matrix with the Majorana phases ( $Z = UP$ ), and a complex orthogonal matrix ( $R$ ) as

$$f_{ij}^\nu = \frac{1}{\langle H_f \rangle} Z_{ik}^* \sqrt{m_{\nu_k}} R_{kj} \sqrt{M_{N_j}}. \quad (33)$$

Using this formula,  $(\delta m_D^2)_{23}$  and  $(\delta m_L^2)_{23}$  are given as

$$\begin{aligned} (\delta m_D^2)_{23} &= \frac{1}{2(4\pi)^2} e^{-i(\varphi_{d_2} - \varphi_{d_3})} \frac{(3m_0^2 + A_0^2)}{\langle H_f \rangle^2} \\ \log \frac{M_G}{M_{GUT}} &((m_{\nu_\mu} + m_{\nu_\tau})(M_{N_2} - M_{N_3}) \cos 2\theta_i \\ &+ (m_{\nu_\mu} - m_{\nu_\tau})(M_{N_2} + M_{N_3}) \cosh 2\theta_i \\ &- 2i\sqrt{m_{\nu_\mu} m_{\nu_\tau}} ((M_{N_2} - M_{N_3}) \sin \phi \sin 2\theta_r \\ &- (M_{N_2} + M_{N_3}) \cos \phi \sinh 2\theta_i), \end{aligned} \quad (34)$$

$$\begin{aligned} (\delta m_L^2)_{23} &= \frac{1}{2(4\pi)^2} \frac{(3m_0^2 + A_0^2)}{\langle H_f \rangle^2} \\ &((m_{\nu_\mu} + m_{\nu_\tau})(\overline{M}_{N_2} - \overline{M}_{N_3}) \cos 2\theta_r \\ &+ (m_{\nu_\mu} - m_{\nu_\tau})(\overline{M}_{N_2} + \overline{M}_{N_3}) \cosh 2\theta_i \\ &- 2i\sqrt{m_{\nu_\mu} m_{\nu_\tau}} ((\overline{M}_2 - \overline{M}_3) \sin \phi \sin 2\theta_r \\ &- (\overline{M}_2 + \overline{M}_3) \cos \phi \sinh 2\theta_i) \end{aligned} \quad (35)$$

with  $\overline{M}_{N_i} = M_{N_i} \log M_G/M_{N_i}$ . Here, we use

$$R = \begin{pmatrix} \cos(\theta_r + i\theta_i) & \sin(\theta_r + i\theta_i) \\ -\sin(\theta_r + i\theta_i) & \cos(\theta_r + i\theta_i) \end{pmatrix}, \quad (36)$$

$$X = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} e^{\phi_M} & \\ & 1 \end{pmatrix}, \quad (37)$$

assuming a maximal mixing for the atmospheric neutrino.  $\phi_M$  is the Majorana phase for the light neutrinos. If the right-handed neutrino masses are hierarchical ( $M_{N_3} \gg M_{N_2}$ ), the correlation is good as [25]

$$\frac{(\delta m_D^2)_{23}}{(\delta m_L^2)_{23}} \simeq e^{-i(\varphi_{d_2} - \varphi_{d_3})} \frac{\log \frac{M_G}{M_{GUT}}}{\log \frac{M_G}{M_{N_3}}}. \quad (38)$$

Thus, it is important to check this GUT relation in the lepton and hadron flavor physics for confirmation of the SUSY GUT with the right-handed neutrinos.

## 5. Mercury EDM in the SUSY GUTs

The Belle experiment in the KEK  $B$  factory reported recently that the CP asymmetry in  $B_d \rightarrow \phi K_s$  ( $S_{\phi K_s}$ ) is  $-0.96 \pm 0.50_{-0.11}^{+0.09}$ , and  $3.5\sigma$  deviation from the Standard-Model (SM) prediction  $0.731 \pm 0.056$  is found [9]. The CP violation in  $B_d \rightarrow \phi K_s$  is sensitive to the new physics since  $b \rightarrow s\bar{s}s$  is a radiative process [26]. In fact, the SUSY models may predict a sizable deviation of the CP violation in  $B_d \rightarrow \phi K_s$  from the SM prediction. If the right-handed bottom and strange squarks have a sizable mixing in the SUSY GUTs with the right-handed neutrinos, the gluon-penguin diagram may give a non-negligible contribution to  $b \rightarrow s\bar{s}s$  in a broad parameter space where the contribution to  $b \rightarrow s\gamma$  is subdominant. Nowadays,  $B_d \rightarrow \phi K_s$  in the SUSY models is studied extensively.

However, the correlation between the CP asymmetry in  $B_d \rightarrow \phi K_s$  ( $S_{\phi K_s}$ ) and the chromoelectric dipole moment (CEDM) of strange quark ( $d_s^C$ ) is strong in the SUSY models with the right-handed squark mixing [12]. In typical SUSY models, the left-handed squarks also have flavor mixing due to the top-quark Yukawa coupling and the CKM mixing (see Eq. (32)), and the left-handed bottom and strange squark mixing is as large as  $\lambda^2 \sim 0.04$ . When both the right-handed and left-handed squark mixings between the second and third generations are non-vanishing, the CEDM of the strange quark is generated. Since  $S_{\phi K_s}$  and  $d_s^C$  may have a strong correlation in the SUSY models with the right-handed squark mixing, the constraint on  $d_s^C$  by the measurement of the EDM of  $^{199}\text{Hg}$  limits the gluon-penguin contribution from the right-handed squark mixing to  $S_{\phi K_s}$  [12].

Before deriving the constraint on the bottom and strange squark mixing, we discuss the EDM of the nuclei. The EDMs of the diamagnetic atoms, such as  $^{199}\text{Hg}$ , come from the CP-violating nuclear force by pion or eta meson exchange. The quark CEDMs,

$$H_{\text{eff}} = \sum_{q=u,d,s} d_q^C \frac{i}{2} g_s \bar{q} \sigma^{\mu\nu} T^A \gamma_5 q G_{\mu\nu}^A, \quad (39)$$

generate the CP-violating meson-nucleon cou-

pling, and the EDM of  $^{199}\text{Hg}$  is evaluated in Ref. [27] as

$$d_{\text{Hg}} = -3.2 \times 10^{-2} e \times (d_d^C - d_u^C - 0.012 d_s^C). \quad (40)$$

The chiral perturbation theory implies that  $\bar{s}s$  in the matrix element of nucleon is not suppressed, and it leads to non-vanishing contribution from the CEDM of the strange quark. The suppression factor in front of  $d_s^C$  in Eq. (40) comes from the eta meson mass and the CP-conserving coupling of the eta meson and nucleon. From current experimental bound on  $d_{\text{Hg}}$  ( $d_{\text{Hg}} < 2.1 \times 10^{-28} e \text{ cm}$ ) [28]

$$e|d_d^C - d_u^C - 0.012 d_s^C| < 7 \times 10^{-27} e \text{ cm}. \quad (41)$$

If  $d_d^C$  and  $d_u^C$  are negligible in the equation,

$$e|d_s^C| < 7 \times 5.8 \times 10^{-25} e \text{ cm}. \quad (42)$$

The neutron EDM should also suffer from the CEDM of the strange quark. However, it is argued in Ref. [29] that the Peccei-Quinn symmetry suppresses it. In the paper the QCD sum rule is adopted to evaluate the neutron EDM. Thus, it does not include the see quark contribution to the neutron EDM, and it is expected that the Peccei-Quinn symmetry decouples the CEDM of the strange quark from the neutron EDM in this evaluation. There is no reliable calculation for the see quark contribution at present. If the “standard” loop calculation of the neutron EDM in the chiral Lagrangian is reliable, the current experimental bound on the neutron EDM may give a comparable constraint on the CEDM of the strange quark.

In the SUSY models, when the left-handed and right-handed squarks have mixings between the second and third generations, the CEDM of the strange quark is generated by a diagram in Fig. 2(a), and it is enhanced by  $m_b/m_s$ . Using the mass insertion technique,  $d_s^C$  is given up to the QCD correction as

$$\begin{aligned} e d_s^C &\simeq e \frac{\alpha_s m_{\tilde{g}}}{4\pi m_{\tilde{q}}^2} \left( -\frac{11}{30} \right) \\ &\times \text{Im} \left[ (\delta_{LL}^{(d)})_{23} (\delta_{LR}^{(d)})_{33} (\delta_{RR}^{(d)})_{32} \right] \\ &= -4.0 \times 10^{-23} \sin \theta e \text{ cm} \left( \frac{m_{\tilde{q}}}{500 \text{ GeV}} \right)^{-3} \end{aligned} \quad (43)$$

$$\times \left( \frac{(\delta_{LL}^{(d)})_{23}}{0.04} \right) \left( \frac{(\delta_{RR}^{(d)})_{32}}{0.04} \right) \left( \frac{\mu \tan \beta}{5000 \text{ GeV}} \right) \quad (44)$$

in a limit of  $m_{\tilde{q}}/m_{\tilde{g}} = 1$ . The mass insertion parameters  $(\delta_{LL}^{(d)})_{23}$ ,  $(\delta_{RR}^{(d)})_{32}$ , and  $(\delta_{LR}^{(d)})_{33}$  are

$$\begin{aligned} (\delta_{LL}^{(d)})_{23} &= \frac{(m_{\tilde{d}_L}^2)_{23}}{m_{\tilde{q}}^2}, \quad (\delta_{RR}^{(d)})_{32} = \frac{(m_{\tilde{d}_R}^2)_{32}}{m_{\tilde{q}}^2}, \\ (\delta_{LR}^{(d)})_{33} &= \frac{m_b (A_b - \mu \tan \beta)}{m_{\tilde{q}}^2}, \end{aligned} \quad (45)$$

and  $\theta = \arg[(\delta_{LL}^{(d)})_{23} (\delta_{LR}^{(d)})_{33} (\delta_{RR}^{(d)})_{32}]$ . In the typical SUSY models,  $(\delta_{LL}^{(d)})_{23}$  is  $O(\lambda^2) \simeq 0.04$ . From this formula, it is obvious that the right-handed squark mixing or the CP-violating phase should be suppressed. For example, for  $m_{\tilde{q}} = 500 \text{ GeV}$ ,  $\mu \tan \beta = 5000 \text{ GeV}$ , and  $(\delta_{LL}^{(d)})_{23} = 0.04$ ,

$$|\sin \theta (\delta_{RR}^{(d)})_{32}| < 5.8 \times 10^{-4}. \quad (46)$$

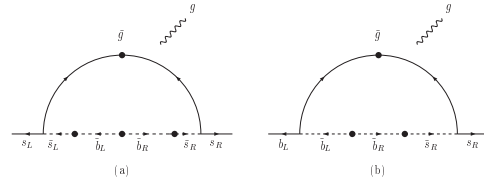


Figure 2. a) The dominant diagram contributing to the CEDM of the strange quark when both the left-handed and right-handed squarks have mixings. b) The dominant SUSY diagram contributing to the CP asymmetry in  $B_d \rightarrow \phi K_s$  when the right-handed squarks have mixing.

In the SUSY SU(5) GUT with the right handed neutrinos, the tau neutrino Yukawa coupling induces the right-handed down-type squark mixing between the second and third generations radiatively as

$$(\delta m_D^2)_{23} \simeq -\frac{2}{(4\pi)^2} e^{-i(\varphi_{d2} - \varphi_{d3})} X_{32} X_{33}^*$$

$$\times \frac{m_{\nu_\tau} M_N}{\langle H_f \rangle^2} (3m_0^2 + A_0^2) \log \frac{M_G}{M_{GUT}}, \quad (47)$$

when the hierarchical neutrino Yukawa coupling and  $U \simeq X$  are assumed. This correction depends on the GUT generic phase. From this equation, the CEDM of the strange quark is larger than the experimental bound when  $M_{N_\tau}$  is larger than about  $10^{(12-13)}$  GeV and  $(\varphi_{d_2} - \varphi_{d_3})$  is of the order of 1. This means that the measurement of the EDM of  $^{199}\text{Hg}$  atom is very sensitive to the right-handed neutrino sector in the SUSY SU(5) GUT.

The current experimental bound on the EDM of  $^{199}\text{Hg}$  atom is determined by the statistics, and the further improvement is expected [28]. Also, it is argued recently in Ref. [30] that the measurement of the deuteron EDM may improve the bound on the CP-violating nuclear force by two order of magnitude. If it is realized, it will be a stringent test on the SUSY models with the right-handed squark mixing, such as the SUSY GUTs.

Now let us discuss the correlation between  $d_s^C$  and  $S_{\phi K_s}$  in the SUSY models with the right-handed squark mixing. The right-handed bottom and strange squark mixing may lead to the sizable deviation of  $S_{\phi K_s}$  from the SM prediction by the gluon-penguin diagram, especially for large  $\tan\beta$ . The box diagrams with the right-handed squark mixing also contribute to  $S_{\phi K_s}$ , however, they tend to be sub-dominant and do not derive the large deviation of  $S_{\phi K_s}$  from the SM prediction. Thus, we neglect the box contribution in this article for simplicity.

The effective operator, which induces the gluon-penguin diagram by the right-handed squark mixing, is

$$H_{\text{eff}} = -C_8^R \frac{g_s}{8\pi^2} m_b \bar{s} \sigma^{\mu\nu} T^A P_L b G_{\mu\nu}^A. \quad (48)$$

When the right-handed squarks have the mixing, the dominant contribution to  $C_8^R$  is supplied by a diagram with the double mass insertion of  $(\delta_{RR}^{(d)})_{32}$  and  $(\delta_{RL}^{(d)})_{33}$  (Fig. 3(b)). Especially, it is significant when  $\mu \tan\beta$  is large. The contribution of Fig. 2(b) to  $C_8^R$  is given up to the QCD correction as

$$C_8^R = \frac{7\pi\alpha_s}{30m_b m_{\tilde{q}}} (\delta_{LR}^{(d)})_{33} (\delta_{RR}^{(d)})_{32}. \quad (49)$$

in a limit of  $m_{\tilde{q}}/m_{\tilde{g}} = 1$ . Comparing Eq. (43) and Eq. (49), a strong correlation between  $d_s^C$  and  $C_8^R$  is derived as

$$d_s^C = -\frac{m_b}{4\pi^2} \frac{11}{7} \text{Im} [(\delta_{LL}^{(d)})_{23} C_8^R] \quad (50)$$

up to the QCD correction [12]. The coefficient 11/7 in Eq. (50) changes from 3 to 1 for  $0 < x < \infty$ .

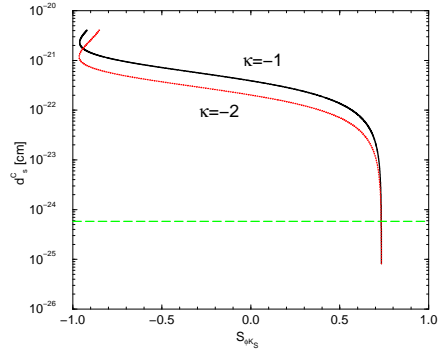


Figure 3. The correlation between  $d_s^C$  and  $S_{\phi K_s}$  assuming  $d_s^C = -m_b/(4\pi^2) \text{Im}[(\delta_{LL}^{(d)})_{23} C_8^R]$ . Here,  $(\delta_{LL}^{(d)})_{23} = -0.04$  and  $\arg[C_8^R] = \pi/2$ .  $\kappa$  comes from the matrix element of chromomagnetic moment in  $B_d \rightarrow \phi K_s$ . The dashed line is the upperbound on  $d_s^C$  from the EDM of  $^{199}\text{Hg}$  atom.

In Fig. 3, the correlation between  $d_s^C$  and  $S_{\phi K_s}$  is presented. Here  $d_s^C = -m_b/(4\pi^2) \text{Im}[(\delta_{LL}^{(d)})_{23} C_8^R]$  is assumed up to the QCD correction. Here, we take  $(\delta_{LL}^{(d)})_{23} = -0.04$ ,  $\arg[C_8^R] = \pi/2$  and  $|C_8^R|$  corresponding to  $10^{-5} < |(\delta_{RR}^{(d)})_{32}| < 0.5$ .  $\kappa$  comes from the matrix element of chromomagnetic moment in  $B_d \rightarrow \phi K_s$ , and  $\kappa = -1.1$  in the heavy-quark effective theory [31]. Since  $\kappa$  may suffer from the large hadron uncertainty, we take  $\kappa = -1$  and  $-2$ . From this figure, it is found that the deviation of  $S_{\phi K_s}$  from the SM prediction due

to the gluon-penguin contribution should be tiny when the constraint on  $d_s^C$  in Eq. (42) is applied.

Finally, we discuss the  $\tau \rightarrow \mu\gamma$  in the SUSY SU(5) GUT with the right-handed neutrinos. In the last section we show the strong correlation between  $(\delta m_D^2)_{23}$  and  $(\delta m_L^2)_{23}$  up to the GUT generic phases in this model. Thus, the constraint on the strange quark CEDM gives a bound on the prediction of  $Br(\tau \rightarrow \mu\gamma)$  smaller than the future sensitivity in the super  $B$  factories when the GUT generic phase is of the order of one.

## 6. Summary

In this article we reviewed the third generation flavor violation in the SUSY seesaw model and SUSY SU(5) GUT with the right-handed neutrinos. After discovering neutrino oscillation, the flavor violation induced the neutrino Yukawa coupling becomes important. Now the  $B$  factories starts to access interesting region for  $\tau \rightarrow \mu(e)\gamma$ . It may give a new information about structure of the seesaw mechanism. Also, we show new stringent constraint from the Mercury EDM on the SUSY models with the right-handed squark mixing, such as the SUSY SU(5) GUT with the right-handed neutrinos, since the Mercury EDM is sensitive to the strange quark CEDM. This constraint implies that the deviation of  $S_{\phi K_s}$  from the SM prediction, which is observed in the Belle experiment, should be suppressed in the SUSY GUT with right-handed neutrinos. Sensitivity to the CP-violating nuclear force may be improved furthermore by about two orders in the measurement of deuterium EDM, and this may be a big impact on SUSY GUT.

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